Student No: ....

Barker College

2012
TRIAL
HIGHER SCHOOL CERTIFICATE

Mathematics Extension 1

ANSWER SHEET

Section I – Multiple Choice
Choose the best response and fill in the response oval completely.

Start Here →

1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○
5.	A ○	B ○	C ○	D ○
6.	A ○	B ○	C ○	D ○
7.	A ○	B ○	C ○	D ○
8.	A ○	B ○	C ○	D ○
9.	A ○	B ○	C ○	D ○
10.	A ○	B ○	C ○	D ○

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Student Number: ………………………………….

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2012
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Mathematics
Extension 1

Staff Involved:

Friday 10th August

- **RMH***
- **GPF***
- **BJB**
- **VAB**
- **GIC**
- **PJR**
- **BHC**

Number of copies: 90

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11 - 14.

Total marks – 70

Section I **Pages 2 - 5**

10 marks

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II **Pages 6 - 10**

60 marks

- Attempt Questions 11 - 14
- Allow about 1 hours 45 minutes for this section

Section I — Multiple Choice

Attempt Questions 1 - 10

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. The x-coordinate of the point which divides the interval joining $A(3,1)$ and $B(-1, 5)$ externally in the ratio 4 : 3 is:

(A) -13 (B) $\frac{5}{7}$ (C) 17 (D) 15

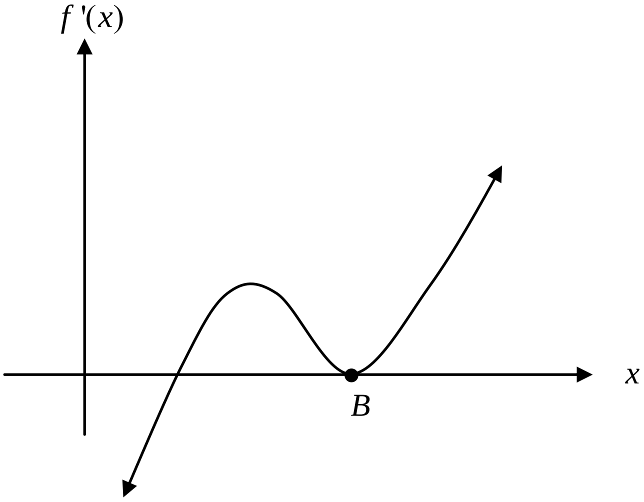
2. $\lim_{x \to 0} \frac{\tan 3x}{2x}$ is equal to :

(A) 0 (B) 1 (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

3. $3\left(\sin^{-1} x + \cos^{-1} x\right)$ is equal to :

(A) 1 (B) $\frac{3\pi}{2}$ (C) 3 (D) 3π

4.



NOT TO SCALE

The gradient function of $y = f(x)$ is shown above. On the curve $y = f(x)$, B would be a :

(A) maximum turning point

(B) point of inflexion

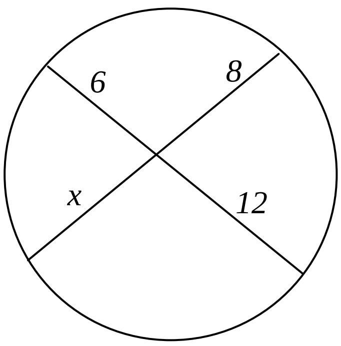
(C) horizontal point of inflexion

(D) minimum turning point

5. Given that $f^{-1}(x) = \dfrac{2x}{x-1}$, $f(x)$ would have the equation :

(A) $\dfrac{x}{x+2}$ (B) $\dfrac{y}{y-2}$ (C) $\dfrac{2y}{y-1}$ (D) $\dfrac{x}{x-2}$

6.



NOT TO SCALE

The value of x is:

(A) 9 (B) 4 (C) 5.5 (D) 1

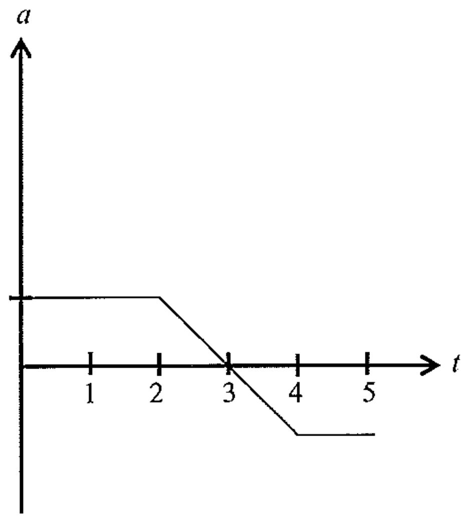
7. A particle moves in simple harmonic motion such that $v^2 = -4(x-5)(x+1)$,

where v is velocity in m/s.

Maximum acceleration of this particle happens when:

(A) $x = 5$ and -1 (B) $x = 2$ (C) $x = -5$ and 1 (D) $x = 0$

8. The acceleration-time graph of a particle is shown below.

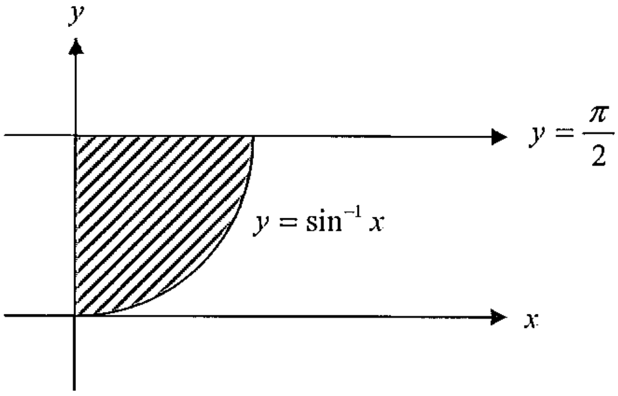


NOT TO SCALE

The time(s) when the particle has a maximum velocity is :

(A) $t = 2$ (B) $t = 3$ (C) $t = 4$ (D) $0 < t < 2$

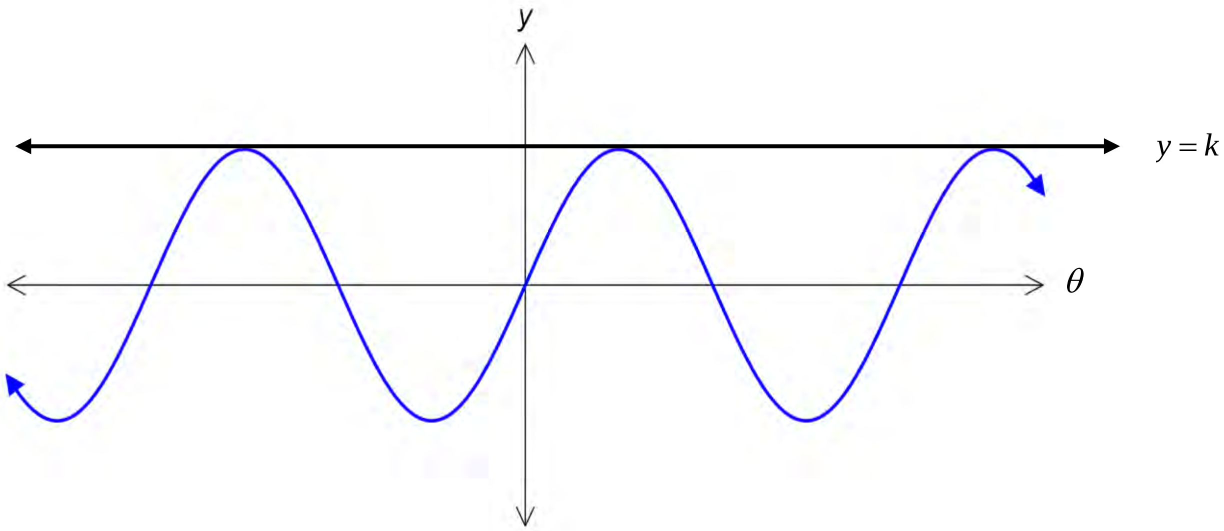
9. The area bounded by the y-axis, $y = \frac{\pi}{2}$ and $y = \sin^{-1} x$ is shaded in the diagram below.



The value of this area, in square units is:

(A) $\frac{\pi}{2} - 1$ (B) 1 (C) $\frac{\pi}{2}$ (D) $1 - \frac{\pi}{2}$

10. Part of the curve $y = 2\sin 2\theta$ is drawn below.



The horizontal line $y = k$ is also drawn as shown above.

The general solution of the intersection of these two functions is:

(A) $\frac{n\pi}{2} + (-1)^n \frac{\pi}{4}$ (B) $2n\pi \pm \frac{\pi}{4}$ (C) $n\pi + (-1)^n \frac{\pi}{2}$ (D) $\frac{n\pi}{2} + (-1)^n \frac{\pi}{2}$

End of Section I

Question 11 (15 marks) **[START A NEW BOOKLET]** **Marks**

(a) Solve $\dfrac{x}{3-2x} < 1$. **4**

(b) Evaluate $\displaystyle\int_{-1}^{1} \frac{dx}{\sqrt{4-x^2}}$, leaving your answer in exact form. **3**

(c) Evaluate $\displaystyle\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \cos^2 2x\, dx$, leaving your answer in exact form. **4**

(d) Using the substitution $x = u - 2$, or otherwise, find $\displaystyle\int \frac{x+1}{\sqrt{(x+2)^3}}\, dx$. **4**

End of Question 11

Question 12 (15 marks) **[START A NEW BOOKLET]** **Marks**

(a) Prove by induction that

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

for all $n \geq 1$, where n is an integer. **3**

(b) Find the coefficient of x^2 in the expansion of $\left(\frac{x^4}{2} + \frac{2}{x}\right)^8$. **3**

(c) Consider the function $f(x) = 3\sin^{-1} x$.

(i) State the domain and range of the function. **2**

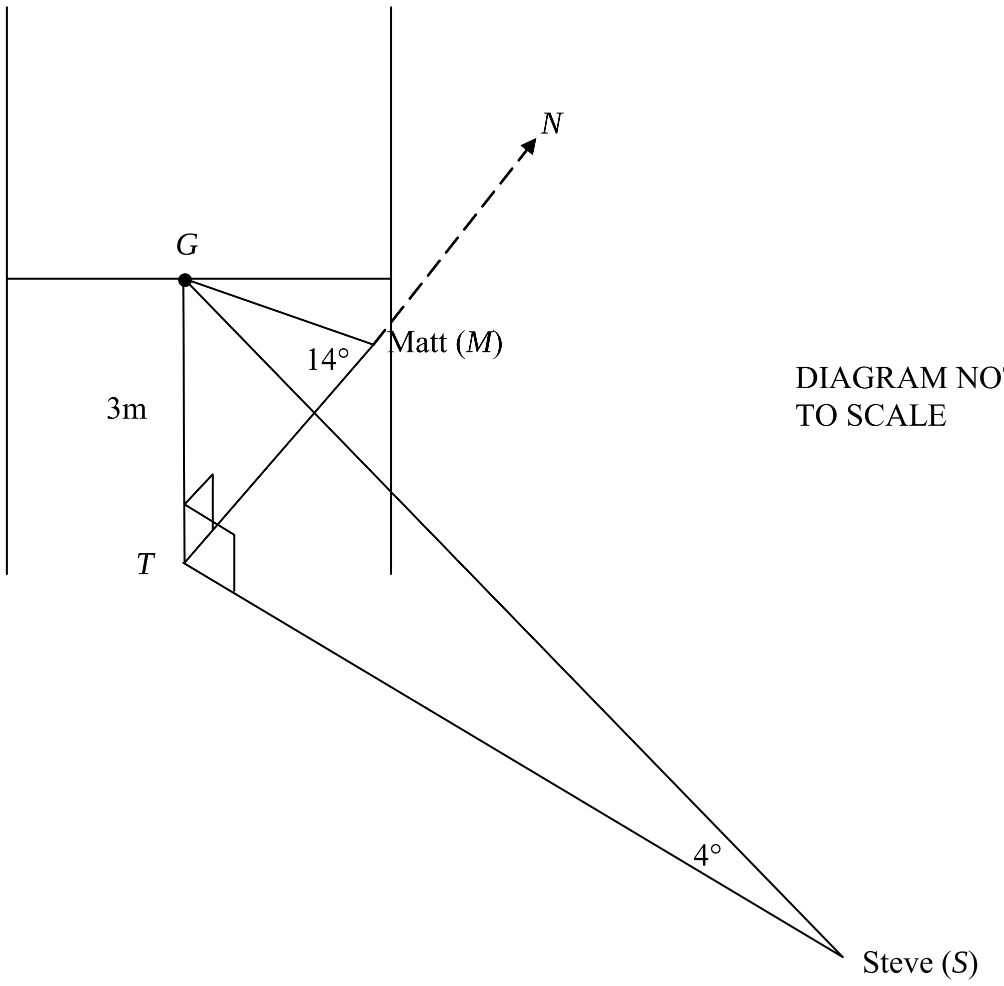
(ii) Draw a neat sketch of the function.
Clearly label all essential features. **1**

(iii) Find the gradient of the tangent to the function $f(x) = 3\sin^{-1} x$ **2**

at the point $\left(\frac{1}{2}, \frac{\pi}{2}\right)$.

Question 12 continues on page 8

(d)



Matt (M) and Steve (S) operate ground-level cameras at a rugby match. The cameras are aimed at the centre of the cross-bar of the goal post (G) which is 3 metres above the ground.
Matt is directly north of the centre of the goal post.
The centre of the goal post is on a bearing of $302°$ from Steve.
The cameras operated by Matt and Steve are set at angles of elevation of 14° and 4° respectively.

(i) Show that $\angle MTS = 122°$. **1**

(ii) Calculate the distance between the two cameras, to the nearest centimetre. **3**

End of Question 12

Question 13 (15 marks) **[START A NEW BOOKLET]** **Marks**

(a) (i) Express $8\cos\theta - 6\sin\theta$ in the form $R\cos(\theta + \alpha)$ where α is in degrees. **2**

(ii) Hence, or otherwise, find the solutions of the equation
$8\cos\theta - 6\sin\theta = 5$ for $0^o \le \theta \le 360^o$. **3**

(b) A particle moves in such a way that its displacement, x cm, from the origin at any time is given by the function $x = 2 + \cos^2 t$ where t is in seconds.

(i) Show that acceleration is given by $\ddot{x} = 10 - 4x$. **2**

(ii) Find the centre of the motion. **1**

(iii) Find the amplitude of the motion. **1**

(c) A ball is kicked on level ground to just clear a fence 2 m high and 40 m away. The initial velocity is 30 m/s and the angle of projection is α.

The displacement equations are

$x = 30t\cos\alpha$ and $y = -5t^2 + 30t\sin\alpha$ (DO NOT PROVE THESE).

(i) Show that $y = \dfrac{-x^2}{180}\sec^2\alpha + x\tan\alpha$. **2**

(ii) Hence, or otherwise, find the angles of projection that allow the ball to just clear the fence. Answer to the nearest degree. **4**

End of Question 13

Question 14 (15 marks) **[START A NEW BOOKLET]** **Marks**

(a) $P\left(2p, p^2\right)$ and $Q\left(2q, q^2\right)$ are two points on the parabola $x^2 = 4y$.
The chord PQ subtends a right angle at the origin.

(i) Show that $pq = -4$. **1**

(ii) If M is the midpoint of PQ, find the locus of M. **2**

(b) Consider the function $f(x) = \dfrac{x+2}{x^2+1}$.

(i) Find the points where the curve crosses the x-axis and y-axis. **1**

(ii) Find the x-coordinates of any stationary points, and without finding the second derivative, determine their nature. **3**

(iii) Describe the behaviour of $y = f(x)$ as $x \to \pm\infty$. **1**

(iv) Sketch the curve $y = f(x)$ using an appropriate scale and showing all the information above. Label the axes and any critical points. **1**

(c) The acceleration of a particle moving along a straight path is given by

$\ddot{x} = -\dfrac{e^x + 1}{e^{2x}}$, where x is in metres.

Initially the particle is at the origin with a velocity of 2 m/s.

(i) show that $v = e^{-x} + 1$. **3**

(ii) Find the equation of the displacement, x, in terms of t. **3**

End of Question 14
End of Paper

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1\,; \quad x \neq 0, \quad \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

2012 Yr 12 Extension 1 Trial HSC

Multiple Choice

① $\left(\dfrac{x_2 k + x_1 l}{k+l}, \quad\right)$

$= \left(\dfrac{-1\times -4 + 3\times 3}{-4+3}, \quad\right)$

$= \left(\dfrac{4+9}{-1}, \quad\right)$

$= (-13, 0)$ Ⓐ

② $\lim\limits_{x \to 0} \dfrac{3}{2} \times \dfrac{\tan 3x}{3x}$

$= \dfrac{3}{2}$ Ⓒ

③ $(\sin^{-1} x + \cos^{-1} x) = \dfrac{\pi}{2}$

$\therefore \dfrac{3\pi}{2}$ Ⓑ

④ + 0 +

/ − /

Ⓒ

⑤ $x = \dfrac{2y}{y-1}$

$xy - x = 2y$

$xy - 2y = x$

$y(x-2) = x$ Ⓓ

$y = \dfrac{x}{x-2}$

⑥ $8x = 72$

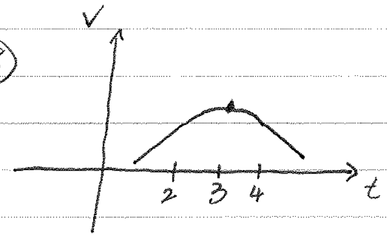
$x = 9$ Ⓐ

⑦ Max acceleration @ $v=0$

$\therefore x = 5, -1$

Ⓐ

⑧



Ⓑ

⑨ $x = \sin y$

$\int_0^{\pi/2} \sin y \, dy$

$= \left[-\cos y\right]_0^{\pi/2}$

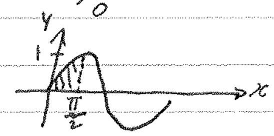
$= -\cos\dfrac{\pi}{2} + \cos 0$

$= 0 + 1$

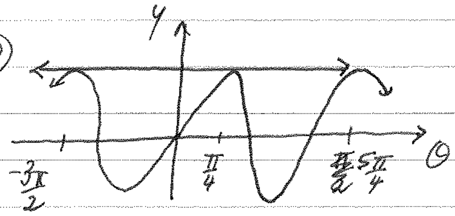
$= 1$ Ⓑ

OR

$A = \int_0^{\frac{\pi}{2}} \sin x \, dx$



⑩



Period = π

Ⓐ

n	0	1	2
$\frac{n\pi}{2} + (-1)^n \frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{\pi}{4}$	$\frac{5\pi}{4}$

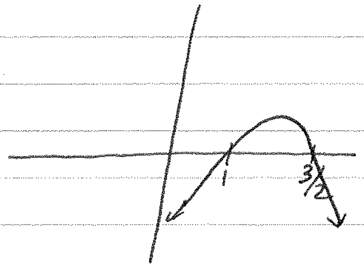
Qu 11 (a) $\frac{(3-2x)^2 x}{(3-2x)} < 1(3-2x)^2$

$x(3-2x) < (3-2x)^2$

$x(3-2x) - (3-2x)^2 < 0$

$(3-2x)[x-(3-2x)] < 0$

$(3-2x)(3x-3) < 0$



$x < 1 \quad , \quad x > 3/2$

(b) $\left[\sin^{-1}\frac{x}{2}\right]_{-1}^{1} = \frac{\pi}{6} - -\frac{\pi}{6}$

$= \pi/3$

(c) $\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

$\frac{1}{2}\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} (1+\cos 4x)\,dx = \frac{1}{2}\left[x + \frac{1}{4}\sin 4x\right]_{\pi/12}^{\pi/4}$

$= \frac{1}{2}\left[\left(\frac{\pi}{4} + \frac{1}{4}\times 0\right) - \left(\frac{\pi}{12} + \frac{1}{4}\sin\frac{\pi}{3}\right)\right]$

$= \frac{1}{2}\left(\frac{\pi}{4} - \frac{\pi}{12}\times\frac{\sqrt{3}}{2}\right)$

$= 1/2\left(\frac{\pi}{4} - \frac{\pi\sqrt{3}}{24}\right)$

(d) $u = x+2$ $\quad \int \frac{u-1}{u^{3/2}}\,du$

$\frac{du}{dx} = 1$

$du = dx \quad = \int \left(u^{-1/2} - u^{-3/2}\right) du$

$= \frac{u^{1/2}}{1/2} - \frac{u^{-1/2}}{-1/2} + c$

$= 2\sqrt{u} + \frac{2}{\sqrt{u}} + c \quad = 2\sqrt{x+2} + \frac{2}{\sqrt{x+2}} + c$

Qn 12

(a) Prove true for $n=1$

$LHS = 1^2 = 1$

$RHS = \frac{1}{6} \times 1 \times (1+1) \times (2+1)$

$= \frac{1}{6} \times 1 \times 2 \times 3$

$= 1$

$\therefore$ True for $n=1$

Assume true for $n=k$

$1^2 + 2^2 + \ldots + k^2 = \frac{1}{6} k(k+1)(2k+1)$

Prove true for $n=k+1$

$LHS = 1^2 + 2^2 + \ldots + k^2 + (k+1)^2$

$RHS = \frac{1}{6} k(k+1)(2k+1) + (k+1)^2$

$= \frac{k+1}{6}\left[k(2k+1) + 6(k+1)\right]$

$= \frac{k+1}{6}(2k^2 + k + 6k + 6)$

$= \frac{k+1}{6}(2k^2 + 7k + 6)$

$= \frac{k+1}{6}(2k+3)(k+2)$

$= \frac{1}{6}(k+1)((k+1)+1)(2(k+1)+1)$

$= S_{k+1}$

If statement true for $n=k$, then true for $n=k+1$.
Since true for $n=1$, then true for ~~the~~ $n=2$
& as true for $n=2$, then true for $n=3$ & so on.
$\therefore$ True for all $n \geq 1$ (where n is an integer)

Qu 12 (Continued)

(b) $^8C_r \left(\frac{x^4}{2}\right)^{8-r}\left(\frac{2}{x}\right)^r$

$= {^8C_r}\left(\frac{1}{2}\right)^{8-r} \cdot 2^r \cdot x^{32-5r}$

$32 - 5r = 2$ $\quad\therefore$ Coefficient

$30 = 5r$ $\quad = {^8C_6} \times \left(\frac{1}{2}\right)^2 \times 2^6$

$\therefore r = 6$ $\quad = 448$

OR

$(1+x)^8 = 1 + 8x + 28x^2 + 56x^3 + 70x^4 + 56x^5 + 28x^6 + 8x^7 + 1x^8$

$\left(\frac{x^4}{2}+\frac{2}{x}\right)^8 = \left(\frac{x^4}{2}\right)^8 + 8\left(\frac{x^4}{2}\right)^7\left(\frac{2}{x}\right) + 28\left(\frac{x^4}{2}\right)^6\left(\frac{2}{x}\right)^2 + \ldots$

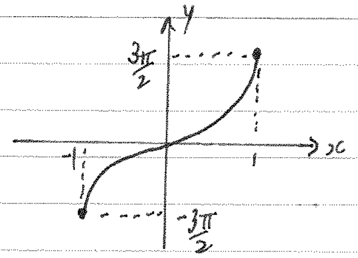
$x^{32} \xrightarrow{-5} x^{27} \xrightarrow{-5} x^{22} \xrightarrow{-5}$

$28\left(\frac{x^4}{2}\right)^2\left(\frac{2}{x}\right)^6 = \frac{28 \times 64}{4} x^2 \quad \therefore 448$

(c) i) D: $-1 \le x \le 1$

R: $-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$

ii)



Qu 12 (Continued)

iii) $f'(x) = \frac{3}{\sqrt{1-x^2}}$

$f'(\tfrac{1}{2}) = \frac{3}{\sqrt{1-1/4}}$

$= 3 \div \frac{\sqrt{3}}{2}$

$= \frac{6}{\sqrt{3}}$

$= 2\sqrt{3}$

Eqn of tangent @ $\left(\frac{1}{2}, \frac{\pi}{2}\right)$

$y - \frac{\pi}{2} = \frac{6}{\sqrt{3}}\left(x - \frac{1}{2}\right)$

$\sqrt{3}y - \frac{\pi\sqrt{3}}{2} = 6x - 3$

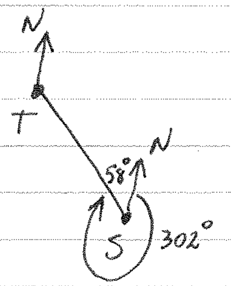
OR

$y - \frac{\pi}{2} = 2\sqrt{3}(x - 1/2)$

$y = 2\sqrt{3}x - \sqrt{3} + \frac{\pi}{2}$

$y = 2\sqrt{3}x - \left(\sqrt{3} - \frac{\pi}{2}\right)$

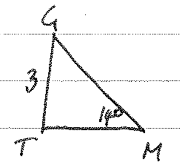
(d) i)



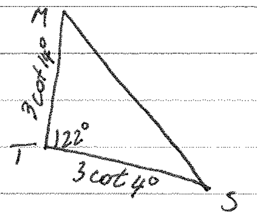
$\angle MTS = 180 - 58$ (cointerior $\angle$'s in // lines)

$= 122°$

ii)



$MT = 3\cot 14°$



$x^2 = (3\cot 14°)^2 + (3\cot 4°)^2 - 2(3\cot 14°)(3\cot 4°)\cos 122°$

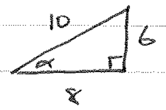
$\doteqdot 2532.46$

$x \doteqdot 50.32\,m$

Qu 13

a) i) $R\cos(\theta+\alpha) = R[\cos\theta\cos\alpha - \sin\theta\sin\alpha]$

$= 10\left(\cos\theta\left(\frac{8}{10}\right) - \frac{6}{10}\sin\theta\right)$



$\therefore \cos\alpha = \frac{8}{10}$

$\alpha = 36°52'$ $\therefore 10\cos(\theta + 36°52')$

ii) $10\cos(\theta + 36°52') = 5$

$\cos(\theta + 36°52') = 0.5$

$\theta + 36°52' = 60°$ or $300°$

$\therefore \theta = 23°08'$ or $263°8'$

b) i) $x = 2 + (\cos t)^2$ $\cos^2 t = x - 2$

$\frac{dx}{dt} = -2\sin t\cos t$

$= -\sin 2t$

$\frac{d^2x}{dt^2} = -2\cos 2t$

$= -2(2\cos^2 t - 1)$

$= -4(x-2) + 2$

$= -4x + 8 + 2$

$= 10 - 4x$

OR

$\frac{dx}{dt} = -2\sin t\cos t$

$\frac{d^2x}{dt^2} = -2\sin t(-\sin t) + (-2\cos t)\cos t$

$= 2\sin^2 t - 2\cos^2 t$

$= 2(\sin^2 t - \cos^2 t)$

$= 2(1 - \cos^2 t - \cos^2 t)$

$= 2(1 - 2\cos^2 t)$

$= 2 - 4\cos^2 t$

$= 2 - 4(x-2)$

$= 2 - 4x + 8$

$= 10 - 4x$

ii) $a = 0$

$0 = 10 - 4x$

$\therefore x = 2.5$ (centre of motion)

iii) $0 \le \cos^2 t \le 1$

$\therefore 2 \le x \le 3$

amplitude $= \frac{1}{2}$ cm

OR

$v = 0$

$0 = -\sin 2t$

$2t = 0, \pi, 2\pi, \ldots$

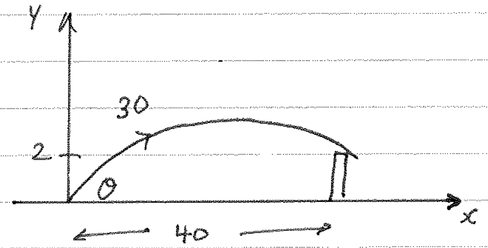
$t = 0, \frac{\pi}{2}, \pi, \ldots$

$t = 0$, $x = 3$ cm $\therefore$ amplitude

$t = \frac{\pi}{2}$, $x = 2$ cm $= \frac{1}{2}$ cm

Qu 13 Continued

c)



i) $t = \dfrac{x}{30\cos\alpha}$ $\therefore y = \dfrac{-5x^2}{900\cos^2\alpha} + \dfrac{30x\sin\alpha}{30\cos\alpha}$

$y = \dfrac{-x^2\sec^2 x}{180} + x\tan\alpha$

ii) $2 = \dfrac{-40^2\sec^2\alpha}{180} + 40\tan\alpha$

$360 = -40^2\sec^2\alpha + 7200\tan\alpha$

$360 = -40^2(\tan^2\alpha+1) + 7200\tan\alpha$

$-9 = 40(\tan^2\alpha+1) - 180\tan\alpha$

$0 = 40\tan^2\alpha - 180\tan\alpha + 49$

$\tan\alpha = \dfrac{180 \pm \sqrt{180^2 - 4(40)(49)}}{2(40)}$

$= \dfrac{180 \pm \sqrt{24560}}{80}$

$\therefore \alpha = 76^\circ 38'$, $16^\circ 14'$ $16^\circ 14' < \alpha < 76^\circ 38'$

$\therefore \alpha = 16^\circ, 77^\circ$

Qu 14

a) i) $M_{OP} \times M_{OQ} = -1$

$$\frac{p^2-0}{2p-0} \times \frac{q^2-0}{2q-0} = -1$$

$$\frac{p}{2} \times \frac{q}{2} = -1$$

$$\boxed{pq = -4}$$

ii) $M = \left(\frac{2p+2q}{2}, \frac{p^2+q^2}{2} \right)$

$= \left(p+q, \frac{p^2+q^2}{2} \right)$

$y = \frac{1}{2}\left((p+q)^2 - 2pq\right)$

$= \frac{1}{2}\left(x^2 - 2x - 4\right)$

$y = \frac{1}{2}x^2 + 4$

b) i) $0 = \frac{x+2}{x^2+1}$ (x-intercept)

$\therefore x = -2$ $(-2,0)$

$y = \frac{0+2}{0+1}$ (y-intercept)

$= 2$ $(0,2)$

Question 14 Continued

ii) $f'(x) = \dfrac{(x^2+1)(1) - (x+2)(2x)}{(x^2+1)^2}$

$= \dfrac{-x^2 - 4x + 1}{(x^2+1)^2}$

$f'(x) = 0$ when $-x^2 - 4x + 1 = 0$

$0 = x^2 + 4x - 1$

$x = \dfrac{-4 \pm \sqrt{16 - 4(1)(-1)}}{2}$

$x = \dfrac{-4 \pm \sqrt{20}}{2}$

$= -2 \pm \sqrt{5}$

x	-5	$-2-\sqrt{5}$	0	$-2+\sqrt{5}$	1
$f'(x)$	$-4/26^2$	0	1	0	$-4/4$

Min Max

iii) $\lim_{x \to \infty} \left(\dfrac{x+2}{x^2+1} \right) = \lim_{x \to \infty} \left(\dfrac{\frac{x}{x^2} + \frac{2}{x^2}}{\frac{x^2}{x^2} + \frac{1}{x^2}} \right)$

$= \dfrac{0}{1}$

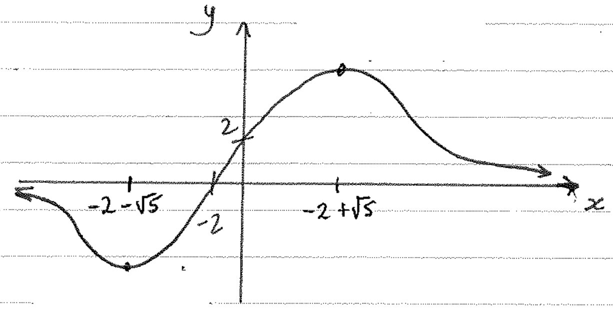
$= 0^+$

$x \to \infty,\ y \to 0^+$

$x \to -\infty,\ y \to 0^-$

Question 14 Continued

iv)



(c) i) $a = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$\frac{1}{2}v^2 = \int\left(-e^{-x} - e^{-2x}\right)dx$

$= e^{-x} + \frac{1}{2}e^{-2x} + c$

When $x=0$, $v=2$

$\therefore \frac{2^2}{2} = e^0 + \frac{1}{2}e^0 + c$

$c = \frac{1}{2}$

$\frac{1}{2}v^2 = e^{-x} + \frac{1}{2}e^{-2x} + \frac{1}{2}$

$v^2 = 2e^{-x} + e^{-2x} + 1$

$= e^{-2x} + 2e^{-x} + 1$

$= \left(e^{-x}+1\right)^2$

$\therefore v = \pm\left(e^{-x}+1\right)$

When $x=0$, $v=2$

$\therefore v = e^{-x}+1$

Qn 14 Continued

ii)

$$\frac{dx}{dt} = e^{-x}+1$$

$$\frac{dt}{dx} = \frac{1}{e^{-x}+1}$$

$$t = \int \frac{1}{e^{-x}+1}\,dx$$

$$= \int \frac{e^x}{e^x} \cdot \frac{1}{e^{-x}+1}\,dx$$

$$= \int \frac{e^x}{e^0+e^x}\,dx$$

$$= \log_e\left(1+e^x\right)+c$$

When $t=0$, $x=0$

$$0 = \ln\left(e^0+1\right)+c$$

$$\therefore c = -\ln 2$$

$$\therefore t = \ln\left(e^x+1\right) - \ln 2$$

$$= \ln\left(\frac{e^x+1}{2}\right)$$

$$e^t = \frac{e^x+1}{2}$$

$$2e^t = e^x+1$$

$$e^x = 2e^t - 1$$

$$x = \ln\left(2e^t-1\right)$$